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An Application of Queues to Offensive Support Indirect Fire Weapons Systems

S. Wheeler

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**Land Operations Division
Systems Sciences Laboratory**

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ABSTRACT

This paper presents a general foundation for an analysis of the grade of service provided by delivery systems in the execution of fire missions. Queueing theory is used to develop several queueing models for the delivery systems in field operations. Furthermore, a number of simulations are constructed from these simple queueing models. These simulations extend the concepts introduced earlier with the inclusion of an x and y co-ordinate reference for both the delivery systems and the targets prosecuted by the delivery systems. In these simulations, three methods for the allocation of targets to delivery systems are examined. Results from simulations are analysed in terms of empirical approximations to stationary distributions and average occupancies.

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Executive Summary

The Australian Army's current indirect fire weapons systems capability is founded on delivery systems, operational procedures, ammunition and technologies from the 1960s. The changing nature of warfare to meet threats in urban environments, complex and littoral terrain has necessitated the requirement for the generation of new lethal and non-lethal effects beyond those currently available.

This research supports the Land 17 Artillery Replacement – 105 mm and 155 mm project, identified in the Australian Government's Defence Capability Plan 2004-2014. This project enhances the Australian Army's indirect fire support capability with the replacement and enhancement of the Australian Defence Force's 105 mm Hamel Howitzer and 155 mm M198 Howitzer as these systems reach the projected end of service in 2010.

The paper proposes two techniques for the evaluation and development of new policies guiding the tactical deployment and use of the future Indirect Fire Weapons Systems for the Australian Defence Force. First, established results from queueing theory are used to develop several new queueing models for field operations conducted by Offensive Support. Several simple examples are provided and potential areas for development discussed. Second, a number of simulations applying queueing theory to Offensive Support, are developed. These simulations are used to compare the performance of three alternative command and control architectures for Offensive Support. Results from these simulations are analysed in terms of well defined constructs such as stationary distributions and occupancies per delivery system.

This study shows that queueing theory is a valuable method of quantitative analysis for new methods of prosecuting targets using Offensive Support. Beyond the numerical measurement of performance, the approach taken generates insights into why and how the various models perform as they do through the observation of the delivery systems in the battlefield. It is demonstrated that the proposed model, which is sensitive to both the location of targets and the number of targets yet to be prosecuted by the delivery systems, is more robust, adaptable and able to prosecute targets more quickly than other candidate models. These results and the approaches suggested, when applied to a concrete example in the real world which uses actual data drawn from combat or field trials as input, have the potential to provide feedback and insight for the development of protocols and doctrine in network enabled operations.

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Scott Wheeler completed a Bachelor of Science (Ma. and Comp. Sc.) at the University of Adelaide with a major in Computer Science and Applied Mathematics in 1996. He received a HECS exemption scholarship during his honours year and graduated top of his class in 1997. In 1998 he enrolled in a Ph.D. under an Australian Postgraduate Award and completed his studies in 2002. Scott joined DSTO as a Research Scientist in 2003 and currently works in Land Operations Division.

Scott's research interests include random search algorithms, linear and network optimisation, mathematical programming and operations research, economic modelling and games theory, and stochastic processes.

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1 Introduction

1.1 Introduction

The Command-and-Control aspects of delivery systems in Offensive Support are currently under scrutiny under the Defence Science and Technology Organisation's support to the Land 17 (Government of Australia 2003, pp.122-123), Artillery Replacement - 105mm and 155mm, project. In particular, the integration of digital technologies and the automation of functions performed within a Command-and-Control architecture are potential areas of interest. Despite the increase in attention generated by these innovations, rigorous techniques for an analysis of the performance of alternative Command-and-Control architectures are as yet lacking.

This document presents a general foundation for an analysis of the grade of service provided by delivery systems in the execution of fire missions. A Birth-and-Death process from queueing theory is used to model asymptotic behaviour of the delivery systems. That is, the stationary distributions for the queues are derived in order to analyse a number of issues of interest. Examples of possible applications for such a model include the following:

- *Effects-based Operations* - calculating the minimum number of delivery systems required to obtain a desired effect or grade of service.
- *Logistics* - investigating the ability to sustain supply to delivery systems under various rates of fire or tempo of battle.
- *Network Centric Warfare* - weighing the relative benefit of improved situational awareness in terms of targets prosecuted.
- *Occupancy* - determining the average occupancy per delivery system.
- *Performance* - predicting how many targets can be prosecuted and how many targets are lost due to overloading the system.
- *Procurement* - comparing two or more delivery systems by quantitative analysis of salient features such as rate of fire.
- *Work Load* - measuring the proportion of time delivery systems are engaged at various thresholds (all delivery systems engaged or 75% delivery systems engaged for example).

To explore these concepts, several queueing models for Offensive Support are developed and subsequently extended for simulation. These models provide a means of investigating the boundaries between analytic models and simulation. Through simulation, we address two points. First, we demonstrate an extended application of a mathematically rigorous model. Second, we investigate the boundary between mathematical models and simulation techniques. Results obtained from simulations are analysed in terms of standard measures from queueing theory.

1.2 Overview

Throughout this paper, we make extensive use of two common distributions, namely the Poisson distribution and the Exponential distribution. The Poisson distribution is commonly used in the literature to model the number of random occurrences of some given phenomenon of interest in a specified unit of space or time. In 1838, the Poisson distribution was published for the first time in the seminal work of *Siméon-Denis Poisson*, *Recherches sur la probabilité des jugements en matières criminelles et matière civile*, Poisson (1838). Since then, the Poisson distribution has been used extensively in the literature in mathematics and statistics. Two applications of note are Bortkiewicz's (1898) study of Prussian cavalry deaths due to being kicked by a horse and Clarke's (1946) study of the German V-2 flying-bomb. The Exponential distribution models the time elapsing before a random event occurs. The Exponential distribution is described in most probability and statistics texts, see for example Balakrishnan and Nevzorov (2003). The probability theory in this study is introduced in Appendix A.

In Section 2, we derive two simple queueing models and work through an application of these models for Offensive Support. A Blocked Calls Cleared model is analyzed in Section 2.2 and a Blocked Calls Held with Defections model is analyzed in Section 2.3. These queueing models are well established in the literature. For an alternative derivation of the queueing models used in this paper consult Cooper (1981), Kelly (1979) and Saaty (1961). Following Preston (1975), we formulate these two models using a spatial Birth-and-Death process. Formulations such as these are described in most probability texts, see for example Feller (1968). Many of the classical results used in this study stem from the work of Agner Krarup Erlang, a Danish mathematician and the father of queueing theory and teletraffic modelling, see Erlang (1909; 1917) for his original analysis.

In subsequent sections, an empirical study that extends the concepts we introduced earlier is conducted. This work represents a new and novel application of queueing theory for Offensive Support. Description of the models we simulate is provided in Section 3 with the empirical results for these simulations in Section 4. A discussion of our research is presented in Section 5 and conclusions are stated in Section 6.

2 Queueing Models

In this section, we derive two simple queueing models and work through an application of these models for Offensive Support. Consult Erlang (1917) for the original work on general queueing models and Cooper (1981), Kelly (1979) and Saaty (1961) for more about queues and their applications, including all models derived in this section.

2.1 Birth-and-Death Process

Consider a simple Birth-and-Death process (Preston 1975) $(B(t), t \in \tau)$, for a τ of either \mathbb{R} or \mathbb{R}^+ , on the state space $\mathcal{S} = \mathbb{N}$. That is, a discrete state, continuous-time Markov process with transition rates or intensities $q(i, i + 1)$ and $q(i + 1, i)$, $i \in \mathbb{N}$.

The equilibrium or global balance equations for the Markov process $B(t)$ are

$$\pi_j \sum_{k \in \mathcal{S}} q(j, k) = \sum_{k \in \mathcal{S}} \pi_k q(k, j), \quad \forall j, k \in \mathcal{S}, \quad (1)$$

where $\pi = (\pi_0, \pi_1, \dots)$ denotes the equilibrium distribution, such that

$$\sum_{j \in \mathcal{S}} \pi_j = 1. \quad (2)$$

Note that, for the stationary distribution to exist it is sufficient that $\exists k$ such that $\forall l \geq k$, $\lambda(l)/(\mu(l+1)) \leq 1$, where $\lambda(l)$ and $\mu(l)$ denote the state dependent transition intensities $q(l, l+1)$ and $q(l+1, l)$, $l \in \mathcal{S}$, respectively.

The global balance equations (1) become

$$\begin{aligned} \lambda(0)\pi_0 &= \mu(1)\pi_1, \\ (\lambda(n) + \mu(n))\pi_n &= \mu(n+1)\pi_{n+1} + \lambda(n-1)\pi_{n-1}, \quad \forall n > 0. \end{aligned} \quad (3)$$

Solving the system (3) recursively gives

$$\pi_n = \pi_0 \prod_{l=1}^n \frac{\lambda(l-1)}{\mu(l)}, \quad (4)$$

where the normalizing equation (2) is used to calculate

$$\pi_0 = \left[\sum_{n=0}^{\infty} \prod_{l=1}^n \frac{\lambda(l-1)}{\mu(l)} \right]^{-1}. \quad (5)$$

2.2 A Blocked Calls Cleared M/M/N/N Model

Suppose that, we are told that calls-for-fire are passed to a number N of delivery systems with a Poisson arrival rate $\lambda(n)$. This arrival rate depends on the number n of calls-for-fire in the system in the following way. If there are less than N calls-for-fire in the system then $\lambda(n)$ is a constant λ but if there are already N calls-for-fire in the system then $\lambda(n) = 0$ and the arrivals of additional calls-for-fire are prohibited. Hence,

$$\lambda(n) = \begin{cases} \lambda, & n < N, \\ 0, & n = N. \end{cases} \quad (6)$$

Furthermore, we know the exponentially distributed service rate at the delivery systems, or can at worst estimate this rate from field trials. Assume that each delivery system acts independently at the rate μ and that calls-for-fire are acted upon once only by a single delivery system. Then, the net service rate is

$$\mu(n) = n\mu, \quad n \leq N. \quad (7)$$

This net service rate models a collection of N delivery systems which processes up to and including a total of N calls-for-fire concurrently.

Given $\lambda(n)$, $\mu(n)$, and an acceptable loss probability (grade of service) denoted by p_g , we pose the following question. What is the smallest number of delivery systems necessary to ensure the loss probability is no more than p_g ?

Substitution of equations (6-7) into equations (4-5) gives

$$\pi_n = \begin{cases} \pi_0 \frac{1}{n!} \left(\frac{\lambda}{\mu}\right)^n, & n \leq N, \\ 0, & n > N, \end{cases} \quad (8)$$

and

$$\pi_0 = \left[\sum_{l=0}^N \frac{1}{l!} \left(\frac{\lambda}{\mu}\right)^l \right]^{-1}. \quad (9)$$

The value π_0 is guaranteed to exist for all finite values λ , μ and N .

The proportion of time that all N delivery systems are in operation, and hence cannot service new arrivals, is given by the Erlang B-Formula or Erlang Loss Formula (Erlang 1917)

$$B(N, \lambda/\mu) = \pi_N = \frac{1}{N!} \left(\frac{\lambda}{\mu}\right)^N \left[\sum_{l=0}^N \frac{1}{l!} \left(\frac{\lambda}{\mu}\right)^l \right]^{-1}. \quad (10)$$

This formula arises as a result of the property of a Poisson process that Poisson Arrivals See Time Averages (PASTA) (Ross 2003, p.480). That is, calls-for-fire with Poisson arrivals see the system as if they enter the system at a random instance in time.

Denote the offered load in Erlangs by

$$a = \frac{\lambda}{\mu}. \quad (11)$$

That is, the ratio of the arrival rate to the service rate. The answer to our original question is then given as the solution to

$$\arg \min_N B(N, a) \leq p_g. \quad (12)$$

For example, from Figure 1, we require 2 delivery systems at an offered load of 0.5 and 5 delivery systems at an offered load of 3, to approximately maintain a $p_{0.1}$ of 10%.

Furthermore, we can also calculate the average number of occupied delivery systems or the carried loading as

$$\sum_{l=0}^N l\pi_l = a(1 - B(N, a)). \quad (13)$$

The average occupancy per delivery system is then given by $\bar{L} = a(1 - B(N, a))/N$.

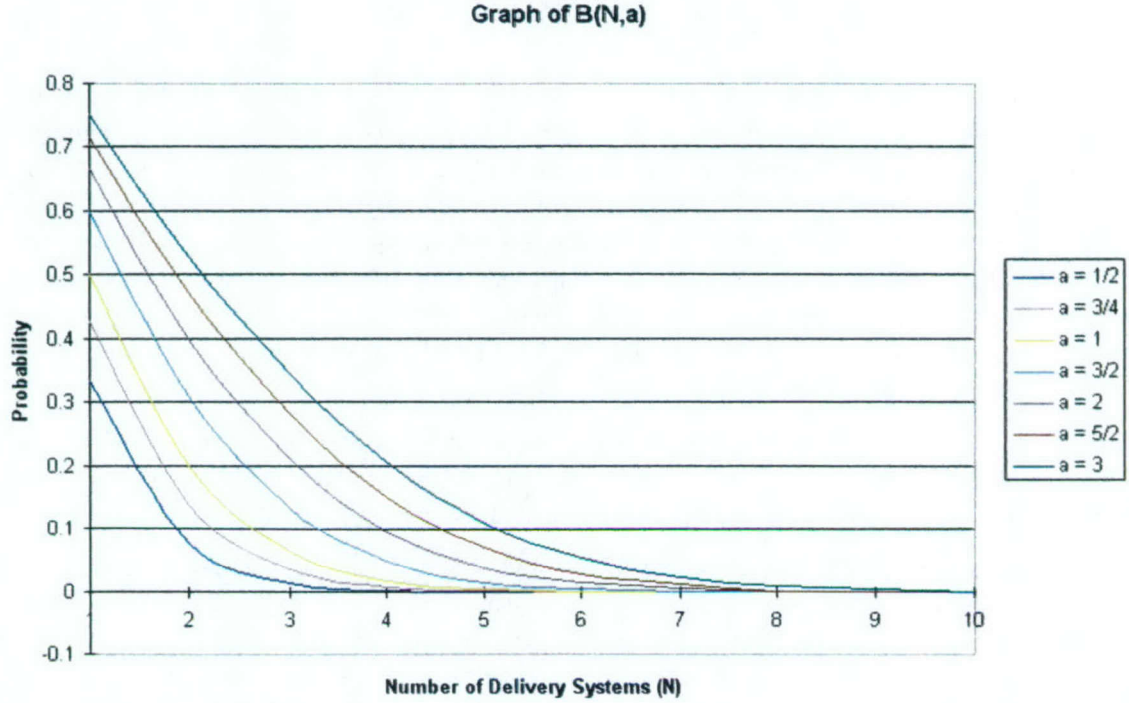


Figure 1: Graph of $B(N, a)$

2.3 A Blocked Calls Held with Defections Model

The model presented in Section 2.2 is easy to explain but is not *per se* an accurate representation of the operation of delivery systems. We propose that calls-for-fire are again sent to a number N of delivery systems. However, if a call-for-fire is not processed immediately, that is if all N delivery systems are busy, then the fire mission is placed into a holding queue. Fire missions in this queue remain in the queue for an exponentially distributed holding time with parameter γ . Hence, time sensitive targets are modelled. If a fire mission in the holding queue is processed before it defects from the system, then it has an exponentially distributed service time with parameter μ .

We have

$$\lambda(n) = \lambda, \quad \forall n, \quad (14)$$

and

$$\mu(n) = \begin{cases} n\mu, & n \leq N, \\ N\mu + (n - N)\gamma, & n > N. \end{cases} \quad (15)$$

Substitution of equations (14–15) into equations (4–5) gives

$$\pi_n = \begin{cases} \pi_0 \frac{a^n}{n!}, & n \leq N, \\ \frac{\pi_0 a^n}{N! \left[N + \frac{\gamma}{\mu}\right] \dots \left[N + (n - N)\frac{\gamma}{\mu}\right]}, & n > N, \end{cases} \quad (16)$$

where a is given by equation (11) and π_0 is calculated by normalization.

3 Simulations

Previous sections establish a mathematical foundation for the application of queueing theory to analyse models for Offensive Support. The potential uses of queueing theory are in no way limited to the simple examples provided in these sections. However, the derivation of closed form solutions to such models may not be necessary in practice. Instead, it is desired that a broad range of potential models be quickly and simply investigated. From such a qualitative study one or more models of interest are investigated in depth using rigorous mathematical techniques. That is, only a comparatively small subset of the potential models of interest are analysed in detail. To identify this subset of models simulation techniques are used.

In this section, we demonstrate an extended application of concepts from queueing theory in a qualitative study. This study simulates several alternative means of command and control for the prosecution of targets identified in streams of calls-for-fire.

A simulation is conducted as follows. A two-dimensional battlefield with dimension 100×100 units is initialised as the area of operations. Four self-propelled delivery systems, labelled 1 through 4 respectively, are placed at the four corners of this area at simulation-time $t = 0$. Calls-for-fire arrive according to a Poisson process with parameter $\lambda = 1/30$. Each call-for-fire is generated with an (x, y) co-ordinate where x and y are uniformly distributed over $0 \dots 100$. Calls-for-fire are processed by the self-propelled delivery systems, assuming that the delivery systems have an infinite buffer, as follows.

- *Model A.* Calls-for-fire are sent to the delivery system closest to the origin of each respective call-for-fire.
- *Model B.* Calls-for-fire are sent to the delivery system closest to the origin of each respective call-for-fire unless that delivery system is already busy in which case the corresponding call-for-fire is sent to the delivery system with shortest queue length, breaking ties in queue length arbitrarily.
- *Model C.* Calls-for-fire c are sent to the delivery system

$$g = \arg \min_{i=1, \dots, 4} (d(i, c) + \alpha l(i)), \quad (17)$$

where $d(i, c)$ denotes the distance between the delivery system i and the call-for-fire c , $l(i)$ denotes the number of calls-for-fire awaiting prosecution in delivery system i 's waiting queue and α denotes a constant input parameter.

Equation (17) is constructed such that α is interpreted as $E[W_q^a]$, where $E[W_q^a]$ denotes an approximation to the expected waiting time of the queue per delivery system. Hence, equation (17) is interpreted as a selection rule choosing one of the four delivery systems based on a comparative estimation of the "business" of each delivery system.

In these simulations, it is important to notice that the interpretation of the queues used to store calls-for-fire by the delivery systems is fundamentally different to that presented in Section 2. By attaching an x and y co-ordinate reference to the calls-for-fire and delivery systems it becomes necessary to differentiate between the calls-for-fire and the delivery systems. Hence, in simulations we interpret the queues as first-in-first-out structures and uniquely identify the four delivery systems using the numbers 1 through 4.

Delivery systems process calls-for-fire in two phases. First, the delivery system g moves to the origin of the call-for-fire c and incurs a movement delay with an exponential distribution, parameter $\mu_m = d(g, c)^{-1}$, where d is defined as in equation (17). Second, the delivery system g engages the target and incurs an engagement delay with an exponential distribution, parameter $\mu_e = 1/20$.

The delivery systems' locations are tracked, partly for simplicity and partly by design, through the origins of the calls-for-fire the systems process. Hence, a movement delay is interpreted as the time it takes for a given delivery system to move from the vicinity of the last call-for-fire processed, or its starting location if the system has yet to process a call-for-fire, to the vicinity of the next call-for-fire such that it is sufficiently close to the origin of the call-for-fire in order to prosecute the target. An additional parameter for the range of the weapons systems is not introduced. Hence, a movement delay is always invoked even when a target is within range of the weapons system prosecuting the target. A more sophisticated model using a step function with a zero movement delay for all targets in weapons' range and a movement delay of μ_m otherwise is certainly possible but is not the main concern of this paper.

4 Empirical Results

4.1 Models A , B and C

Results for queueing models such as those presented in Section 2 are typically analysed in terms of the asymptotic properties of Birth-and-Death processes. For example, a stationary distribution denotes the proportion of time spent in each of the possible states of the queueing system on an infinite time scale and is independent of the initial state of the system. However, any simulation we run is terminated after some designated number of iterations or designated time period. Hence, we approximate the stationary distribution by running a simulation for as long as is practical.

Before simulation, it is necessary to first check that a stationary distribution to the system exists. That is, the number of calls-for-fire in the system does not increase to infinity. Obviously, results from simulations are meaningless in this situation. Such a concern is not as important in an analytic calculation of the stationary distribution because determining the existence of the stationary distribution is an inherent part of its calculation. Refer to Section 2.1 for a sufficiency condition for the existence of the stationary distribution in Birth-and-Death processes with infinite buffers.

We conduct twenty independent simulations of each of the three models A through C from Section 3 over a time scale of 50,000 units per simulation. The value of α in equation (17) is initialised to 45. In each simulation the following values are recorded: the average occupancy of the system per delivery system \bar{L} , the average occupancy of the queue per delivery system \bar{L}^q , the mean waiting time of calls-for-fire in the system \bar{W} and the mean waiting time of calls-for-fire in the queues \bar{W}^q . The mean and standard deviations of these values over the twenty simulations are displayed in Table 1, where the subscripts A , B , and C are used to discriminate between the three models.

Table 1: Simulation results for models A, B and C

	Mean	Std Dev		Mean	Std Dev		Mean	Std Dev
\bar{L}_A	2.983/4	0.345/4	\bar{L}_B	4.026/4	0.294/4	\bar{L}_C	2.743/4	0.268/4
\bar{L}_A^q	1.361/4	0.303/4	\bar{L}_B^q	1.628/4	0.188/4	\bar{L}_C^q	1.155/4	0.214/4
\bar{W}_A	89.260	9.669	\bar{W}_B	116.800	6.744	\bar{W}_C	83.521	7.761
\bar{W}_A^q	40.655	8.663	\bar{W}_B^q	44.696	4.788	\bar{W}_C^q	35.329	6.356

In each simulation, the stationary distribution of the system over the duration of the simulation is also recorded. In measuring the stationary distribution of the system, a delivery system is considered occupied if it is prosecuting a target. This includes both movement and engagement. The average occupancy per delivery system then denotes the average number of calls-for-fire in the system and the average occupancy of the queue per delivery system denotes the average number of calls-for-fire in each of the four queues. The mean waiting time denotes the average amount of time taken by the system in prosecuting a target and the mean waiting time in the queue denotes the average amount of time spent before calls-for-fire are acted upon by one of the four delivery systems.

Figure 2 provides an illustrative comparison of the stationary distributions of the three models averaged over the twenty simulations. The error bars displayed in this figure denote the 95% confidence intervals corresponding to the t -statistic with 19 degrees of freedom.

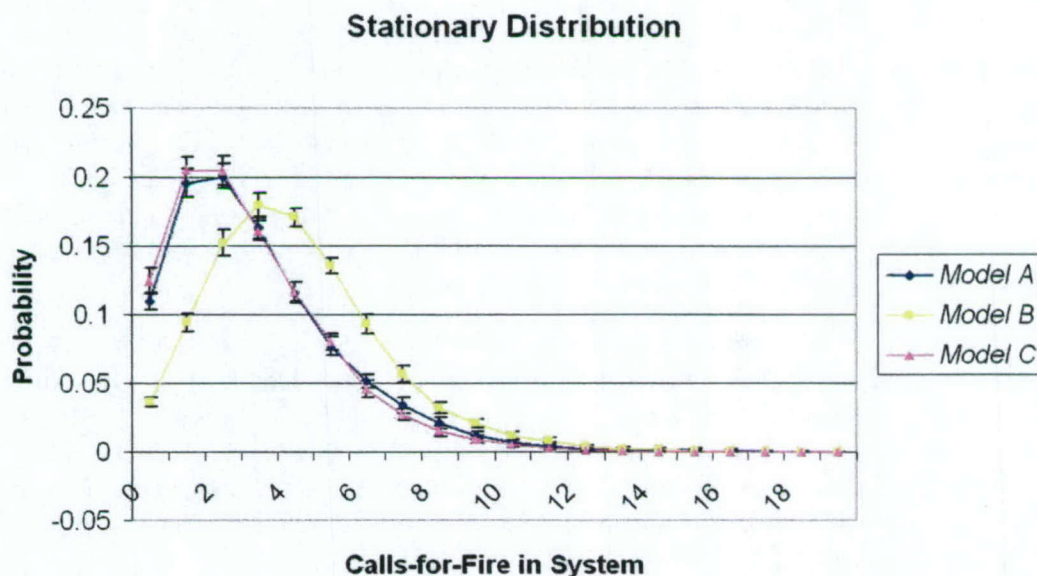


Figure 2: Stationary distributions for models A, B and C

We now statistically test the significance of our results, focusing on the average occupancies of our models. Let \bar{L}_A^T and \bar{L}_C^T denote the true means of the average occupancies of models A and C respectively. Then, to statistically test the hypothesis $H_0 : \bar{L}_C^T = \bar{L}_A^T$ against the alternative hypothesis $H_a : \bar{L}_C^T < \bar{L}_A^T$, we use a two sample t -test with 35 degrees of freedom, see (Moore and McCabe, 1993, p.538). A t -statistic of 2.454 is calculated. This

value corresponds to a probability value p in the range $(0.005, 0.1)$. This means that our null hypothesis H_0 is rejected in favor of the alternative hypothesis H_a at a significance level of $\alpha = 0.1$ but is retained at a significance level of $\alpha = 0.005$. Hence, in rejecting the null hypothesis H_0 at a significance level of $\alpha = 0.1$ we are 99% confident that H_0 is false and that the true mean for the average occupancy of model A exceeds the true mean for the average occupancy of model C . We conclude that, for the particular parametrisation and experimental method defined in this section, model C has statistically fewer calls-for-fire in the system on average than does model A . Similar hypothesis tests comparing the average occupancies of models A and C against the average occupancy of model B are statistically significant with near 100% confidence. In these tests it is almost certain that both models A and C have statistically fewer calls-for-fire in the system on average than model B .

To argue why the results displayed in Table 1 are obtained we consider the behaviours typically exhibited by the three models in a single simulation. Examples of the behaviours of the delivery systems for the three models A , B and C are provided in Figures 3, 4 and 5 respectively. Model A exhibits superior performance to model B . It is observed in Figure 3 that the delivery systems in model A move about the battlefield less than in model B . Note that the mean delay in time invoked by movement exceeds the mean delay in time invoked by engagement for distances greater than 20 units. Hence, it is more efficient to queue calls-for-fire at a delivery system in a local area of operations than it is to allocate calls-for-fire to inactive delivery systems at sufficiently large distances from the origins of the calls-for-fire. As Figure 4 illustrates, the allocation of calls-for-fire to delivery systems on the basis of the length of their queue causes the delivery systems to move erratically across the battlespace. Model C performs slightly better than model A . Although model A performs well, model C has the advantage of allocating some of the calls-for-fire away from busy delivery systems to less busy delivery systems by calculating the comparative estimation of busyness as given in equation (17).

Equation (17) from Model C provides one possible example of a function used to distribute calls-for-fire amongst four delivery systems. The proposed form of the equation, a sum of distance and queue length with a co-efficient α to scale the relative contribution the two terms make to the total, is sufficiently simple to be easily modelled. Potentially, we would like to replace constants such as α with parameters which are updated dynamically during simulations using measurements of performances of the delivery systems from the recorded histories of those delivery system. Also, notice that each delivery system knows the distance between itself and the origin of the next call-for-fire in its queue, the distance between the origin of all calls-for-fire in its queue and the expected time it requires to engage targets. Hence, it is possible for each delivery system to work out the expected time until it becomes free, that is, the expected time until it has processed all the current calls-for-fire from its queue. Assuming that the delivery systems communicate between themselves, each delivery system may predict the expected time before any of its neighbors may service any new calls-for-fire. When the time delay for movement is taken into account, each delivery system can then calculate if it is expected that the call-for-fire will be serviced more quickly if the delivery system accepts the call-for-fire onto its own queue, or it is expected that the call-for-fire will be serviced more quickly if the delivery system leaves the call-for-fire to another. Both of the two alternatives proposed above to equation (17) are robust to changes in the battlefield. Ideally, the delivery systems could be provided with

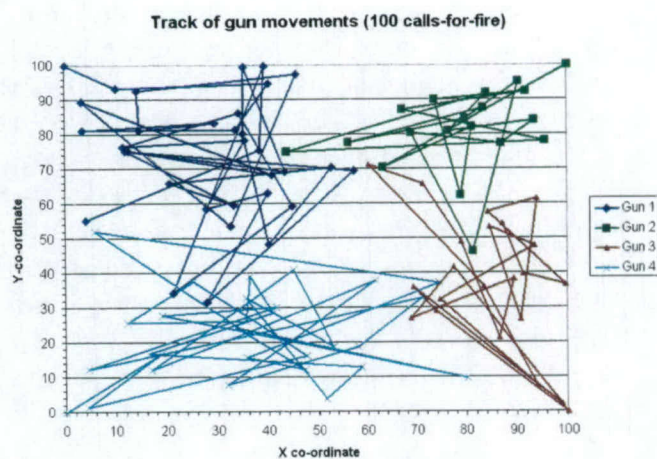


Figure 3: Delivery system (Gun) movements, model A

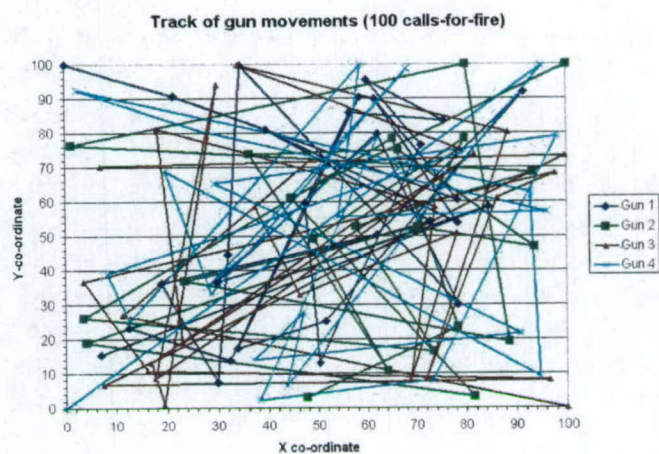


Figure 4: Delivery system (Gun) movements, model B

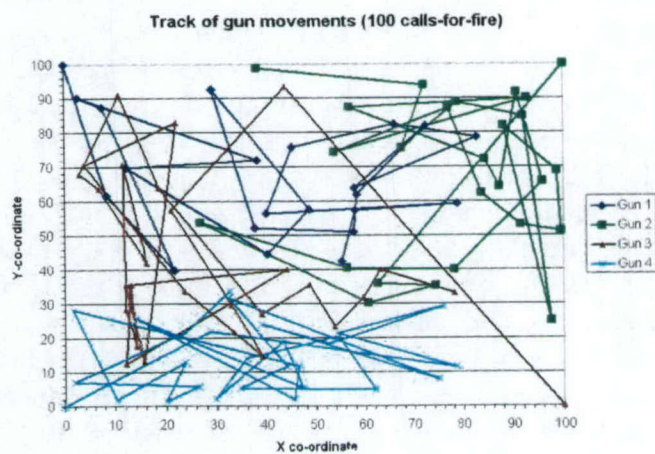


Figure 5: Delivery system (Gun) movements, model C

sophisticated routing algorithms, perhaps modelled on a modified Travelling Salesman problem. Using these algorithms, the delivery systems could select the order that calls-for-fire are processed to minimize the distance travelled between successive calls-for-fire.

4.2 Models \tilde{A} , \tilde{B} and \tilde{C}

The assumption of uniformly distributed x and y target locations made in Section 4.1 is unrealistic. To investigate if the models are susceptible to exploitation by adversaries, in correlating the locations of arrivals to better model a directed enemy attack, we consider the following modifications. First, we change the origin of calls-for-fire so that the origins oscillate between the top left region of the battlefield and the lower right region of the battlefield. That is, if the counter c denotes the c^{th} call-for-fire, then the c^{th} call-for-fire is generated at the (x, y) co-ordinate on the battlefield such that x and y are uniformly distributed over $[1, 35]$ if $c \bmod 2 = 0$ and x and y are uniformly distributed over $[65, 100]$ otherwise. Next, we set the parameter from (17) to $\alpha = 40$, and the arrival rate of calls-for-fire to $\lambda = 1/25$. These values are chosen both for the purpose of demonstration and to suit the way calls-for-fire are now generated. To differentiate between the original three models and the new ones, we denote the new models by \tilde{A} , \tilde{B} and \tilde{C} respectively.

We simulate each of the three models \tilde{A} through \tilde{C} over a time scale of 5,000 units. The empirical results are given in Table 2.

Table 2: Simulation results for models \tilde{A} and \tilde{C}

	Mean	Std Dev		Mean	Std Dev
$\bar{L}_{\tilde{A}}$	3.230/4	1.229/4	$\bar{L}_{\tilde{C}}$	1.961/4	0.402/4
$\bar{L}_{\tilde{A}}^q$	1.772/4	1.124/4	$\bar{L}_{\tilde{C}}^q$	0.513/4	0.216/4
$\bar{W}_{\tilde{A}}$	83.621	27.664	$\bar{W}_{\tilde{C}}$	51.383	8.335
$\bar{W}_{\tilde{A}}^q$	45.841	26.386	$\bar{W}_{\tilde{C}}^q$	13.342	5.159

Results for model \tilde{B} are not provided because a stationary distribution of this model does not exist. That is, the delivery systems are unable to process the stream of calls-for-fire. The length of the queue asymptotically increases to infinity because the movement and engagement times combine to exceed the inter-arrival times of the calls-for-fire. However, it is still possible to gain insight into the behaviour of model \tilde{B} through simulation. An illustrative comparison of the stationary distributions for models \tilde{A} and \tilde{C} averaged over the twenty simulations is presented in Figure 6. The error bars displayed in this figure denote the 95% confidence intervals corresponding to the t -statistic with 19 degrees of freedom.

A t -test conducted on the average number of calls-for-fire in the system, presented in Table 2, yields a t -statistic of 4.391. Hence, with near 100% confidence, it is almost certain that model C has statistically fewer calls-for-fire in the system on average than model A .

Examples of the behaviours of the delivery systems for the three models \tilde{A} , \tilde{B} and \tilde{C} are provided in Figures 7, 8 and Figure 9 respectively. In model \tilde{A} , the delivery systems are allocated calls-for-fire primarily on the distance between themselves and the origin of the calls-for-fire. Hence, we observe that delivery system 1 and delivery system 3 never

Stationary Distribution

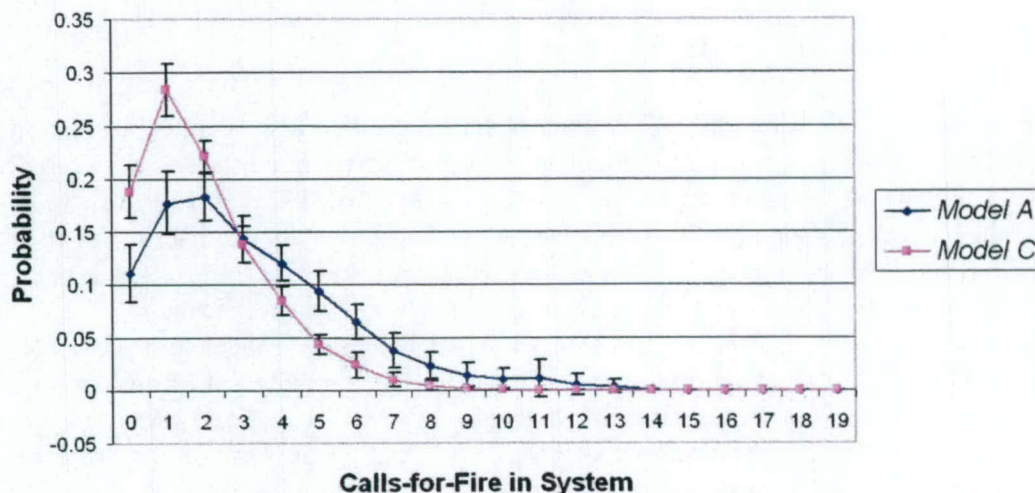


Figure 6: Stationary distributions for models \tilde{A} and \tilde{C}

leave their starting positions and are never utilised. In model \tilde{B} , the delivery systems are allocated calls-for-fire primarily on the length of their queues. Hence, we observe that the delivery systems are continually travelling between the top right and bottom left corners of the battlefield. This results in poor performance of the system but a high utilisation of the delivery systems due to their movement. In model \tilde{C} , delivery system 1 and delivery system 3 move from their starting locations to assist delivery system 2 and delivery system 4 while avoiding the behaviour observed in model \tilde{A} . This results in superior performance of the system.

The concepts used in this section are based on well defined mathematical definitions and not *ad hoc* measures of performance. Stationary distributions, occupancies, and waiting times are widely accepted terms. Furthermore, such an approach is useful both to measure the performance of an entire system and to measure the performance of components of the system. Consider, for example, the effect of introducing laser target designators to forward observers. Few would argue that a laser target designator does not enhance the performance of the forward observer in a measurable way. However, it is necessary that any such measurable improvement be analysed in the context of the system itself. It could be, for example, that any improvement in the performance of the forward observer is negligible compared with the time expended obtaining clear-air and clear-ground clearance (Coombs *et al.* 2004). This is an analogous concept to the observation in this study that the time expended in movement of the four self-propelled delivery systems exceeds that expended in firing upon targets. Furthermore, it is not sufficient to simply state this observation. To determine if laser target designators do in fact provide an improvement across the system, concepts such as stationary distributions, average occupancies, and mean waiting times are needed.

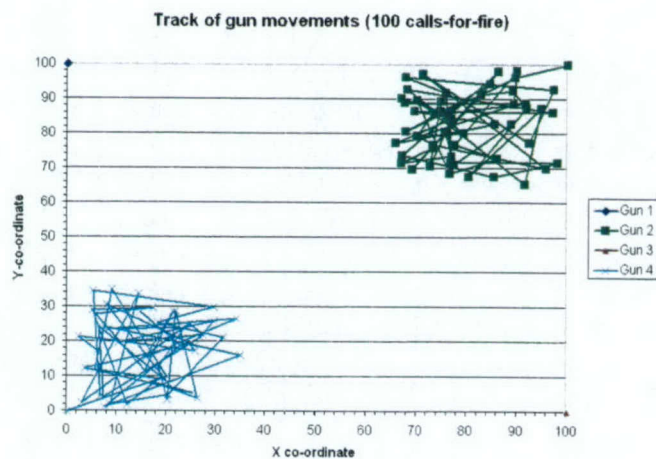


Figure 7: Delivery system (Gun) movements, model \tilde{A}

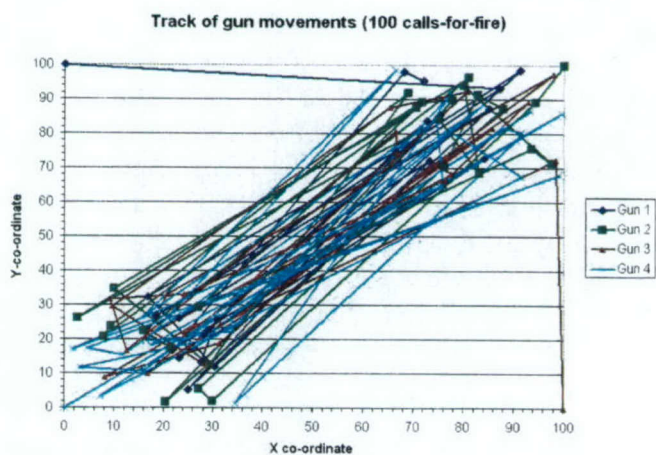


Figure 8: Delivery system (Gun) movements, model \tilde{B}

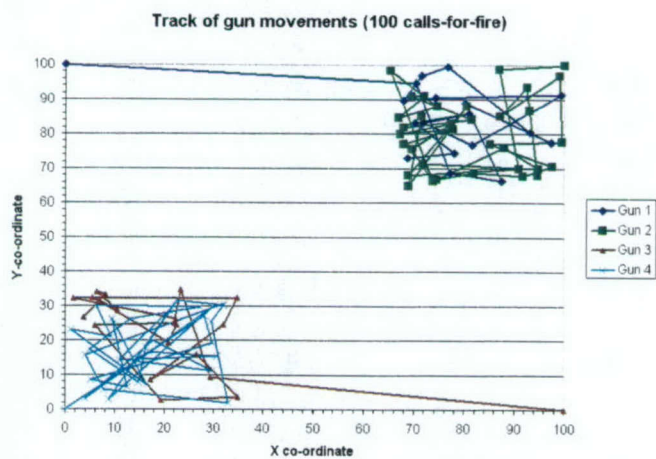


Figure 9: Delivery system (Gun) movements, model \tilde{C}

5 Discussion

We have presented a general framework for the use of queues in an exploration of the impact of alternative Command and Control structures for Offensive Support. Two concepts were examined. First, we posed several models for Offensive Support using queues. Queueing theory *per se* was shown to be a valuable method of quantitative analysis. Second, we used simulation to illustrate an extension of the theory. This approach was shown to be a valuable method to quickly explore a range of potential models when an empirical approximation to an analytic model was acceptable. Furthermore, simulation was demonstrated to be useful in exploring the behaviours of models over finite time spans from given starting states. Hence, short-term behaviours can be studied even when the models have very different long-term behaviours, when the long-term behaviours are known not to exist or when short-term transitions from given starting states are more important than long-term behaviours approaching equilibria.

In this study, no attempt was made to test the sensitivity of our results across a number or different sets of input values. That is, the particular parametrisations explored in this study are but one instance of an infinite number of possible combinations of input values. For example, in Section 3 an arrival rate of $\lambda = 1/30$ was defined. In practice this arrival rate is itself a random variable with distribution F say. Then, an extensive study of a number of instances of the arrival rate, sampling across its distribution F , is required in order to robustly study the performance of our models. Similarly, μ_e , μ_m , and α all require further study. Such a study is better left until actual data from field trials is available. A study of the possibilities for the (x, y) coordinates, as conducted in Section 4.2 for example, is less easily addressed because there is no simple systematic way to define realistic options. To address this problem, a known scenario of interest must first be developed. Potentially this scenario would be based on military wargames, either table-top or virtual, and military field exercises.

The examples provided in this study were indicative of the potential applications of queueing theory rather than definitive. The scope of mathematically tractable applications for queues exceeds that discussed in this paper. For example, each of the following items could have been modelled: battle damage assessment, priority streams of different types of targets, and different types of servers such as close air support. Alternatively, a study of the performance of the self-propelled delivery systems in an environment in which calls-for-fire are densely packed together and an environment in which calls-for-fire are sparse distributed could have been conducted.

Finally, this study is one of three papers investigating the potential uses of mathematics, statistics and simulation in the analysis of models for Offensive Support. This study pushes the boundary of mathematically tractable models towards simulation models that are analysed using well-defined concepts and queueing theory. The second paper (Wheeler and White 2004) explores the use of embedding mathematical models such as simple linear optimisation within a simulation. Analysis of results is performed using techniques from the field of statistics. The last (Wheeler and Ryan 2004) of the three papers builds upon the simple examples provided in the other two papers and compares the approaches employed. In particular, emphasis is placed upon two models that are designed to study the effects of allocating calls-for-fire to delivery systems optimally, based on the solution

to an integer linear program, and the effects of processing calls-for-fire optimally, based on a Traveling Salesman problem.

6 Conclusions

This study has presented two new approaches for measuring the performance of delivery systems in the battlefield. First, the application of two classical queueing models for Offensive Support was discussed. These models were useful in answering questions about the number of delivery systems required to achieve an acceptable grade of service. However, the pure classical queueing models were found to be limited in scope of application because many of the potential problems facing Offensive Support were intractably complex. For such cases, an approach based on simulation was developed. Using simulation, the performance of three new methods for the prosecution of targets was analysed. Of these three methods, one was demonstrated to exhibit superior performance across the scenarios considered. This method, denoted model *C*, was responsive to the distance between the delivery systems and the targets, and the number of targets yet to be prosecuted by each of the delivery systems. Hence, it was better able to gauge the comparative business of the delivery systems and to allocate calls-for-fire to delivery systems more evenly than other proposed methods. This property was maintained even in a scenario specifically designed to stress the methods in which both of the other two exhibited a substantial degradation in performance. We conclude that, within the limits of this study, our proposed model *C* is more robust, adaptable and able to prosecute targets more quickly than the other models considered.

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Appendix A: Probability Theory

A.1 Poisson Distribution

The Poisson distribution (Poisson 1838) is commonly used in the literature to model the number of random occurrences of some given phenomenon of interest in a specified unit of space or time.

Applications of the Poisson distribution include modelling:

- the number of Prussian soldiers killed by a horse kick per year in each of the fourteen Prussian cavalry corps in a twenty year study (Bortkiewicz 1898),
- AT&T's call processing simulator for telephone calls received by inbound call centers (Brigandi *et al.* 1994), and
- the number of German V-2 flying bombs falling on each square mile of London during a German air raid in the early part of the second World War (Clarke 1946).

Let X denote the discrete random variable that counts the number of occurrences of a given phenomenon arising in a block or unit of time of given length. Let this random variable be distributed according to a Poisson distribution with parameter λ . That is, $X \sim P(\lambda)$. The probability of x occurrences is

$$\mathbb{P}(X = x) = \frac{\lambda^x e^{-\lambda}}{x!}, \quad x \in \mathbb{N}. \quad (18)$$

A.2 Exponential Distribution

The Exponential distribution (Balakrishnan and Nevzorov 2003) is commonly used in the literature to model the distance in time or space between randomly generated occurrences of some given phenomenon of interest. Note that the Exponential distribution can also be derived as a special case of the Weibull distribution (Weibull 1939) where $\beta = 1$.

Applications of the Exponential distribution include modelling:

- the time until your next car accident,
- the time until the next terrorist attack on the United States, and
- the distance between spelling mistakes made in a document.

Let T denote the continuous random variable which records the distance in time or space between randomly generated phenomenon. Let this random variable be distributed according to an Exponential distribution with parameter μ . That is, $T \sim EXP(\mu)$. Then, the distribution function for this random variable is given by

$$f(t) = \mu e^{-\mu t}, \quad t \geq 0, \quad (19)$$

and the cumulative distribution function for this random variable is given by

$$F(t) = 1 - e^{-\mu t}, \quad t \geq 0. \quad (20)$$

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